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Nucleon form factors and $O(a)$ Improvement*

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Nucleon form factors have been extensively studied both experimentally and theoretically for many years. We report here on new results of a high statistics quenched lattice QCD calculation of vector and axial-vector nucleon form factors at low momentum transfer within the Symanzik improvement programme. The simulations are performed at three κ and three β values allowing first an extrapolation to the chiral limit and then an extrapolation in the lattice spacing to the continuum limit. The computations are all fully non-perturbative. A comparison with experimental results is made.

1. INTRODUCTION

For many years experiments have been performed with electron–nucleon scattering to obtain information about the structure of the nucleon. Form factors are defined from the general decomposition of the proton, p (or neutron, n) matrix element[1] ($q = p - p'$):

$$\langle \vec{p}, \vec{s} | \widehat{\mathcal{V}}_\mu^{\frac{2}{3}u - \frac{1}{3}d}(\vec{q}) | \vec{p}', \vec{s}' \rangle = \overline{u}(\vec{p}, \vec{s}) \left[\gamma_\mu F_1^p + \sigma_{\mu\nu} \tfrac{q_\nu}{2m} F_2^p \right] u(\vec{p}', \vec{s}').$$

We have $F_1(0) = 1$ as $\mathcal{V}$ is a conserved current, while $F_2(0) = \mu - 1$ measures the anomalous magnetic moment (in magnetons). Usually we define the Sachs form factors:

$$\begin{aligned} G_e(-q^2) &= F_1(-q^2) + \frac{-q^2}{(2m)^2} F_2(-q^2), \\ G_m(-q^2) &= F_1(-q^2) + F_2(-q^2). \end{aligned}$$

*Talk given by R. Horsley at Lat98, Boulder, U.S.A.
[1]We have already re-written everything in euclidean space, so that eg $p = (iE_p, \vec{p})$ and $-q^2 \equiv q^{(\mathcal{M})2} > 0$.

Experiments lead to phenomenological dipole fits:

$$\begin{aligned} G_e^p(-q^2) &\sim \frac{G_m^p(-q^2)}{\mu^p} \sim \frac{G_m^n(-q^2)}{\mu^n} \\ &= 1/\left(1 + \left(-q^2/m_V^2\right)\right)^2, \\ G_e^n(-q^2) &\sim 0, \end{aligned}$$

with $m_V \sim 0.82$ GeV, $\mu^p \sim 2.79$, $\mu^n \sim -1.91$.

Neutrino–neutron scattering, $n\nu_\mu \to p\mu^-$, gives from the charged weak current the axial form factor $g_A(-q^2)$. In addition $g_A(0)$ is also accurately obtained from β-decay, $n \to pe^-\overline{\nu}$. Upon using current algebra this form factor can be related to the matrix element:

$$\langle \vec{p}, \vec{s} | \widehat{\mathcal{A}}_\mu^{u-d}(\vec{q}) | \vec{p}', \vec{s}' \rangle = \overline{u}(\vec{p}, \vec{s}) \left[\gamma_\mu \gamma_5 g_A + i\gamma_5 \tfrac{q_\mu}{2m} h_A \right] u(\vec{p}', \vec{s}').$$

The phenomenological fits are:

$$g_A(q^2) = g_A(0)/\left(1 + \left(-q^2/m_A^2\right)\right)^2,$$

with $g_A(0) = 1.26$, $m_A \sim 1.00$ GeV.

2. THE LATTICE METHOD

Quenched configurations have been generated at $\beta = 6.0$ ($O(500)$, $16^3 \times 32$ lattice) $\beta = 6.2$ ($O(300)$, $24^3 \times 48$ lattice) and $\beta = 6.4$ ($O(100)$, $32^3 \times 48$ lattice), [1]. By forming the ratio of three-to-two point functions, [2]:

$$R_{\alpha\beta}(t,\tau;\vec{p},\vec{q}) = \frac{\langle N_\alpha(t;\vec{p})\mathcal{O}(\tau;\vec{q})\overline{N}_\beta(0;\vec{p}\,')\rangle}{\langle N(t;\vec{p})\overline{N}(0;\vec{p})\rangle} \times$$

$$\left[\frac{\langle N(\tau;\vec{p})\overline{N}(0;\vec{p})\rangle\langle N(t;\vec{p})\overline{N}(0;\vec{p})\rangle\langle N(t-\tau;\vec{p}\,')\overline{N}(0;\vec{p}\,')\rangle}{\langle N(\tau;\vec{p}\,')\overline{N}(0;\vec{p}\,')\rangle\langle N(t;\vec{p}\,')\overline{N}(0;\vec{p}\,')\rangle\langle N(t-\tau;\vec{p})\overline{N}(0;\vec{p})\rangle}\right]^{\frac{1}{2}}$$

$$\propto \langle N_\alpha(\vec{p})|\widehat{\mathcal{O}}(\vec{q})|N_\beta(\vec{p}\,')\rangle,$$

the appropriate matrix elements can be found. (Only the quark line connected part of the 3-point function is considered.) For each β we chose three κ values and a variety of 3-momenta: $\vec{p} = 2\pi/N_s\{\ (0,0,0),\ (1,0,0)\ \}$, $\vec{q} = 2\pi/N_s\{\ (0,0,0), (0,1,0), (0,2,0), (1,0,0), (2,0,0), (1,1,0), (1,1,1), (0,0,1)\ \}$ together with the nucleon either unpolarised or polarised in the y direction. (Some combinations were too noisy to be used though.) After sorting the matrix elements into q^2 classes (defined by q^2 in the chiral limit), 4-parameter fits are made assuming that the form factors are linear in the bare quark mass am_q. $O(a)$ improved Symanzik operators are used:

$$\begin{aligned}
\mathcal{V}^R_\mu &= Z_V(1 + b_V am_q) \times \\
&\quad \left[\bar{\psi}\gamma_\mu\psi + \tfrac{1}{2}ic_V a\partial_\lambda(\bar{\psi}\sigma_{\mu\lambda}\psi)\right], \\
\mathcal{A}^R_\mu &= Z_A(1 + b_A am_q) \times \\
&\quad \left[\bar{\psi}\gamma_\mu\gamma_5\psi + \tfrac{1}{2}c_A a\partial_\mu(\bar{\psi}\gamma_5\psi)\right],
\end{aligned}$$

where Z_V, Z_A, b_V, c_V, c_A (and c_{sw}) have been non-perturbatively calculated by the Alpha collaboration, [3]. All matrix elements thus are correct to $O(a^2)$. We can check Z_V as $\mathcal{V}_\mu$ is a conserved current (ie $F_1(0) = 1$). In Fig. 1 we show a comparison of the two determinations of Z_V. Very good agreement is seen. This is not the case when Wilson fermions are used (see ref. [5]). Finally we note that although we have included the improvement terms in our operators, numerically they seem to have little influence on the value of the matrix element.

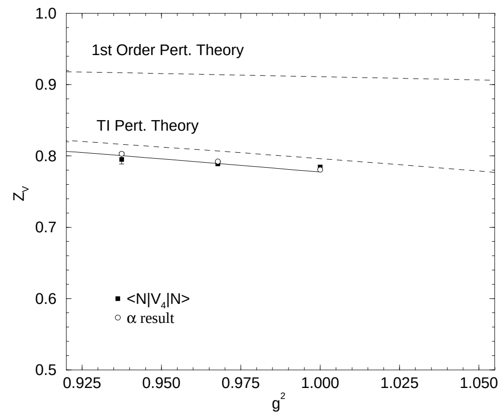

Figure 1. Z_V for improved fermions. Shown is the lowest order perturbation result together with a tadpole-improved version (as given in [4]). The non-perturbative determinations are shown as open circles, [3], and filled squares, this work.

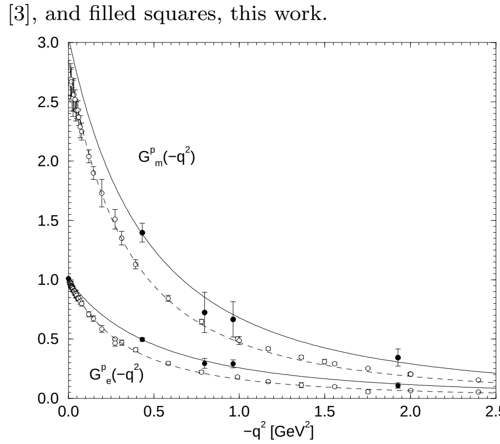

Figure 2. The proton form-factors $G^p_e(-q^2)$ and $G^p_m(-q^2)$ against $-q^2$ showing experimental results (open circles, taken from ref. [6]) and lattice results (filled circles, $\beta = 6.2$ only). The string tension is used to fix the scale as in [4]. All fits are dipole fits.

3. RESULTS

In Fig. 2 we show $G^p_e(-q^2)$ and $G^p_m(-q^2)$ for $\beta = 6.2$ together with experimental results (also plotting the other β values tends to clutter the picture). Making dipole fits gives Fig. 3 for the continuum extrapolation. There seems to be little inclination for m_V to approach the experimental result. (A roughly similar result is obtained from G^p_m, although due to larger error bars the results are more compatible.)

For the axial current we find the results in Figs. 4, 5. The form factor fall-off is again too

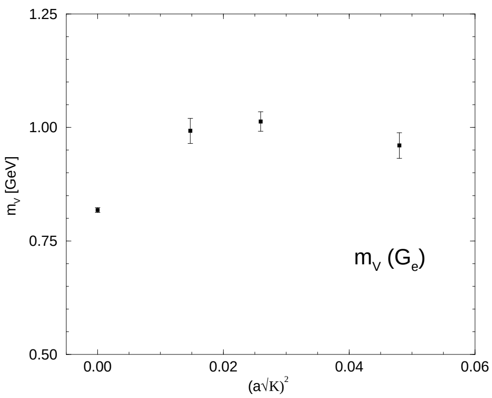

Figure 3. m_V from $\beta = 6.0, 6.2, 6.4$ against a^2. The phenomenological value is also given at $a^2 = 0$.

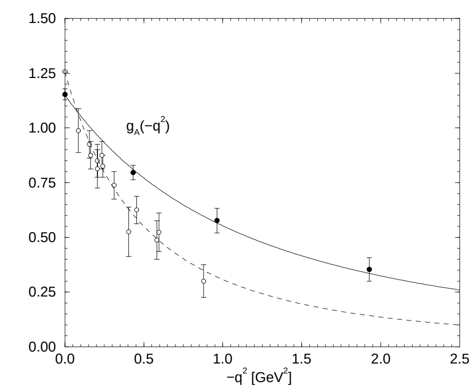

Figure 4. $g_A(-q^2)$ against $-q^2$, notation as in Fig. 2.

soft as m_A is too large. However the important $g_A(0)$ is faring better, see Fig. 6.

4. CONCLUSIONS

We have performed simulations at three β values so that an attempt can be made to take the continuum extrapolation, $a \to 0$. While the lattice dipole masses seem to be too large, $g_A(0)$ is in reasonable agreement with the experimental result. The mass discrepancies may be due to a quenching effect, although only similar simulations using dynamical fermions will be able to answer this.

ACKNOWLEDGEMENTS

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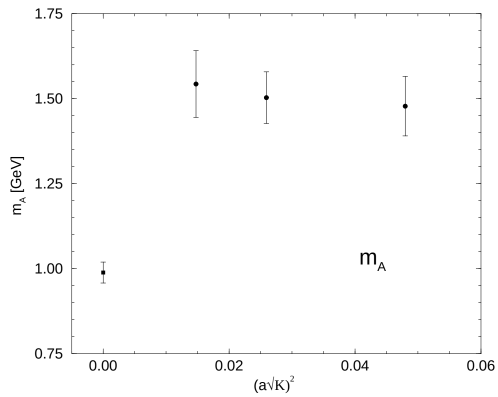

Figure 5. The continuum extrapolation of m_A.

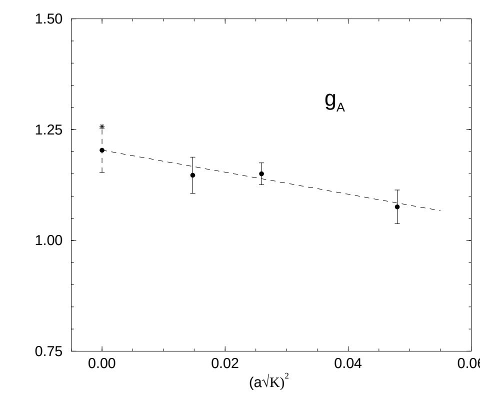

Figure 6. The continuum extrapolation of $g_A(0)$.

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